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Competing effects of point versus columnar defects on the roughening of directed polymers in random media

Igor Arsenin,[1] Timothy Halpin-Healy,[2] and Joachim Krug[3]
[1]*Physics Department, Columbia University, New York, New York 10027*
[2]*Physics Department, Barnard College, New York, New York 10027-6598*
[3]*Institut für Festkörperforschung, Forschungszentrum, D-52425 Jülich, Germany*
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Inspired by recent experiments on cuprate superconductors investigating the enhanced pinning of vortex lines, we have performed transfer matrix studies of directed polymers in random media subject to both point and columnar defects, focusing our attention on the competition between these two different types of disorder. We find that the positional fluctuations in a mixed random medium are larger than in the case of purely columnar or point defects, and show via Flory arguments that the asymptotic behavior is *sub-ballistic*, $x \sim t/(\ln t)^{\psi}$, with $\psi = 2$ in 1+1 dimensions—a value confirmed by our numerics and indicative of a new universality class.

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With the discovery of superconducting ceramic materials, there has been a renaissance of interest in the phenomenology of Abrikosov flux-line lattices, as well as the statistical mechanics of isolated, roughened vortices. Initially, particular attention was paid to the importance of thermal fluctuations—Nelson [1] pointed out that enhanced flux-line wandering would lead to an entangled vortex liquid at magnetic fields slightly above H_{c1}. Nattermann and Lipowsky [2], commenting upon this work, noted that the presence of quenched point defects in these ceramic superconductors would further enhance the wandering, but not fundamentally alter Nelson's conclusions. In addition, these authors were responsible for furnishing some timely publicity to the apparently distant problem of directed polymers in random media (DPRM), just introduced by Kardar and Zhang [3]. Indeed, implicit in their discussion was the assumption that an isolated vortex line in a dirty sample of $YBa_2Cu_3O_7$ was a physical realization of the (2+1)-dimensional DPRM, with its telltale *superdiffusive* wandering exponent. Subsequent work by Fisher [4] and collaborators [5], drawing upon a reformulation of collective pinning theory in the language of disorder-induced roughening [6], addressed the difficult issue of vortex line assemblies in the presence of point disorder and predicted a novel *vortex glass* phase characterized by a nonlinear resistivity. A separate theoretical analysis, appropriate to situations of strongly correlated disorder (i.e., where columnar defects resulting from grain boundaries or, perhaps, forests of screw dislocations provide the dominant pinning mechanism), was proposed by Nelson and Vinokur [7] and has since been labeled the *Bose glass* model by virtue of its origins in Anderson localization effects of bosons in two dimensions [8]. The Bose glass model is closely linked to an experiment by Civale *et al.* [9], who investigated vortex confinement effects due to an artificially created columnar microstructure resulting from high energy ion bombardment of their superconducting sample. As vigorous debate commenced regarding the relevance of these two models, proponents were quick to stress the essential differences between them. In short, while point disorder promotes flux-line wandering and entanglement, correlated disorder inhibits wandering and promotes localization. Since the two theories, as first proposed, entertained rather different limiting cases, it is natural to consider the competing effects of point and columnar defects. In particular, we would like to determine the consequences of this competition, both on the phase diagram of the Abrikosov flux-line lattice, as well as on the disorder-induced roughening of individual vortex lines. Hwa, Nelson, and Vinokur [10] have recently tended to the former, extending the nominal Bose glass theory to include point disorder. The purpose of the present work is to address the latter. We shall do so within the context of the DPRM, where columnar defects are the result of time-independent noise, a matter which was examined as such by Zhang [11] a few years ago using techniques of Mott variable-range hopping. Building on Zhang's efforts and related work [12], Krug and Halpin-Healy [13] have just performed an exhaustive study of the purely columnar DPRM, examining issues of universality at zero and finite temperature. Here, by contrast, we propose a model that interpolates between point and columnar defect limits and discuss the results of a zero-temperature transfer matrix study investigating the competition between these defects on the DPRM. Since we consider isolated vortices, our results are relevant to the limit of low flux-line density, high defect density. Even so, our simulations indicate some surprising, perhaps counterintuitive findings—chief among them that the purely columnar defect fixed point is unstable to the presence of point disorder, being controlled by a new fixed point where the wandering is *sub-ballistic* and governed by strong logarithmic corrections. After examining the geometric and free energy fluctuations of the DPRM, as well as typical trajectories of the directed polymer, we close with a brief discussion of possible experimental ramifications.

In the continuum formulation of our DPRM problem, we consider the configuration of a single directed polymer (i.e., vortex line) in a random medium populated by both point and columnar pins to be specified by its transverse position $\mathbf{x}(t)$ as a function of the longitudinal coordinate

t, which indicates the propagation direction of this elastic line. The partition function governing the statistical mechanics of the p/c DPRM in d transverse dimensions is given by

$$Z(\mathbf{x},t) = \int \mathcal{D}\mathbf{x} \exp\left[-\frac{1}{T}\int dt\left\{\frac{\sigma}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 + V_p(\mathbf{x},t) + V_c(\mathbf{x})\right\}\right], \quad (1)$$

where T is the temperature and σ the line tension which discourages transverse wandering of the directed polymer, while $V_p(\mathbf{x},t)$ and $V_c(\mathbf{x})$ are responsible for the disorder-induced roughening associated with point and columnar defects, respectively. In the simplest scenario these two forms of quenched disorder are assumed to be spatially uncorrelated from defect to defect. The underlying physical picture is manifest—we consider a directed polymer starting at the origin, propagating into the upper half-space $t > 0$, whose transverse positional fluctuations result from a competition between point defects, which continually promote short length scale, but cumulative wandering, and columnar defects, which induce localization effects punctuated by intermittent large-scale migratory jumps to the pins of lower energy, the whole process overseen by the modest effects of an elasticity that discourages any severe deviations from linearity. At finite temperature, the p/c DPRM involves summing over all possible paths weighting each with the appropriate Boltzmann factor, whereas at $T = 0$, the matter becomes one of *global optimization* in which the directed polymer configuration dominating the partition function is the path of overall least energy. In the case $V_c = 0$ (i.e., point defects only), much is known concerning the statistical mechanics of the DPRM; for example, a fluctuation-dissipation theorem [14] reveals that the superdiffusive transverse wandering of the polymer scales as $|x| \sim t^{\zeta=2/3}$, while the energy fluctuations from sample to sample grow as $e_{\rm rms} \sim t^{\theta=1/3}$. In higher dimensions, the DPRM critical indices are not known exactly, but only via numerical work [15], which indicates that $\zeta_{2+1} \approx \frac{5}{8}$, though it is believed that the exponent relation $\theta = 2\zeta - 1$ holds true generally [16].

The zero-temperature p/c DPRM on the square lattice is defined by the recursion relation,

$$E(x,t+1) = \min\{E(x\pm 1,t), E(x,t)\} + \eta(x,t+1) + \varepsilon(x), \quad (2)$$

for the ground-state energy $E(x,t)$ of all directed paths of length t that end at the transverse position x, where $1 \le x \le L$. The point defect energies $\eta(x,t)$ are random numbers drawn uniformly from the interval $[0,p]$, while the columnar defects, properly thought of as running through the vertical bonds of the lattice, have random energies $\varepsilon(x) \in [0,c]$ that are fixed at the start of the calculation. Note that we permit only single transverse steps. This numerically expedient restriction is expected to give rise to an effective line tension σ on large scales. Permitting multistep transverse jumps, while more appropriate to actual vortex line wandering, would not incur any changes in the universal quantities of interest. We employ periodic boundary conditions in the transverse directions [15]. This permits us to simultaneously propagate an ensemble of L locally optimal paths; that is, paths constrained to end at a specified transverse position. In the flux-line problem such a constraint could arise from strong pinning centers at the surface of the sample.

In Fig. 1, we summarize our findings concerning the statistical properties exhibited by the 1+1 p/c DPRM. The data follow from our $T = 0$ numerical transfer matrix calculations done with transverse system size $L = 6000$, and 400 realizations of the mixed random media populated by point and columnar defects. The transverse positional fluctuations are recorded in Fig. 1(a) for different values of p/c, which gauges the relative strengths of the two types of disorder. Similarly, the energy fluctuations of the globally optimal path are recorded in Fig. 1(b). As expected, in the limit of purely *point* disorder, with p/c large, a least squares fit to the data associated with the final thousand steps yields the slopes $\zeta = 0.67 \pm 0.01$ and $\theta = 0.32 \pm 0.01$ for $p/c = 100$. By contrast, at the oppo-

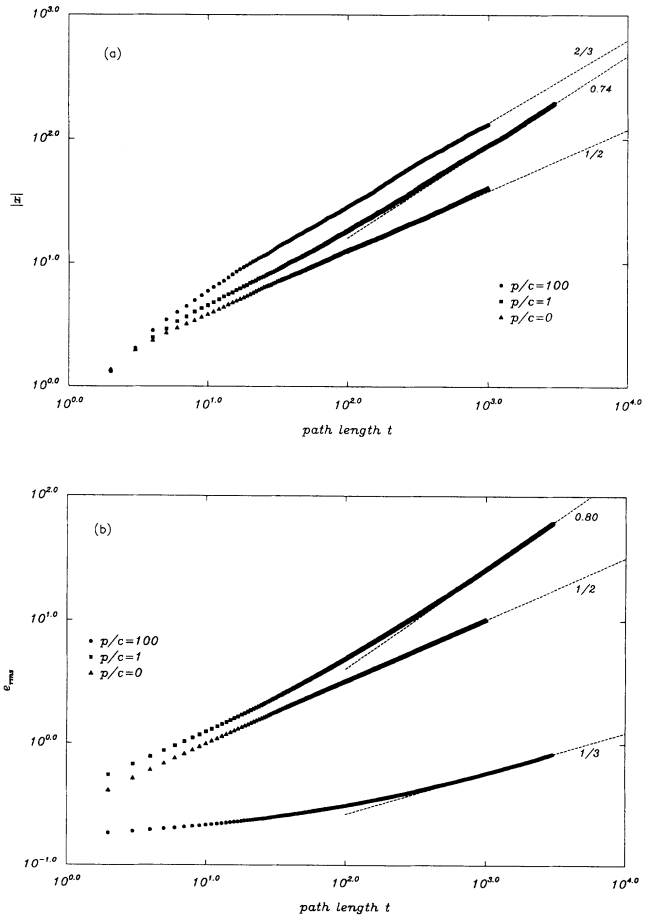

FIG. 1. (a) Positional and (b) energy fluctuations of the directed polymer in random media, subject to the competing effects of point versus columnar defects. The relative strength of the two types of disorder is gauged by the parameter p/c. The slopes of the straight lines are indicated.

site extreme, characteristic of the *columnar* defect fixed point, $p/c = 0$, we find $\zeta = \theta = 0.51 \pm 0.01$. That the wandering and energy fluctuation exponents are identical for the purely columnar DPRM, and that $\zeta = \theta = \frac{1}{2}$ for uniform disorder distributions, has been explained by Krug and Halpin-Healy [13]. Finally, we present illustrative data for the case of a nonzero, but modest value of p/c. The interesting feature immediately apparent from the positional fluctuation plot is that the wandering exponent due to the combined effect of columnar and point defects is clearly greater than it is in the isolated presence of either type of disorder. Indeed, a fit as above produces an effective exponent $\zeta_{\text{eff}} \approx 0.74$, though a careful examination of the data reveals that, unlike the purely point or columnar cases, the DPRM wandering exponent in the mixed random medium is truly effective, never asymptotic and, in fact, *ever-increasing.* This behavior is even stronger for the energy fluctuations, see Fig. 1(b), where $\theta_{\text{eff}} \approx 0.80$, suggesting that the assumed power-law behavior is inappropriate in this matter and that a new fixed point governs the disorder-induced roughening of directed polymers at finite p/c.

Further insight can be gained from a variable-range hopping picture developed in close analogy with the finite-temperature, purely columnar DPRM [13]. We noted above that the point defects encourage local wandering of the polymer, while columnar disorder tends to localize it at some particularly favorable defect. More precisely, the point disorder induces an energy cost $l^{-\tau}$ per unit length, if the positional fluctuations are confined to a transverse scale l, where $\tau = -2 + 2/\zeta$ [17]. In order to localize the polymer in a region of size l, the columnar defect energies in the region therefore have to be of the order $\varepsilon \sim l^{-\tau}$. The probability to find such a region is, in d transverse dimensions,

$$\mathcal{P}(\varepsilon) \sim \varepsilon^{l^d} \sim \exp\left[-\frac{|\ln \varepsilon|}{\varepsilon^{d/\tau}}\right]. \tag{3}$$

These rare, low energy regions act as localization centers for the polymer, in complete correspondence to the Anderson localized states in the finite-temperature, purely columnar DPRM [13]. The resulting conformation of the optimal path can be pictured as follows: from the anchored end point, placed arbitrarily at $x = 0$, the polymer makes a rapid transverse excursion to the most favorable localization center that it can reach, and remains there for most of its extension. From (3), we estimate the columnar energy of the most favorable region within a distance x of the origin to be $\varepsilon_{\min}(x) \sim (d \ln x)^{-\tau/d}$, ignoring higher order logarithmic dependences. The total columnar contribution to the energy of the optimal paths is, therefore, given by the Flory expression:

$$\Phi(x,t) = \bar{\varepsilon} x + (t - x)\varepsilon_{\min}(x), \tag{4}$$

where the first term arises from the initial transverse displacement. Minimizing with respect to x, we obtain $x \sim t/(\ln t)^{\psi}$, with $\psi = 1 + \tau/d$. Using the known value, $\frac{2}{3}$, for the purely point DPRM roughening exponent, we obtain $\tau = 1$ and $\psi = 2$ for the 1+1 p/c DPRM, a value

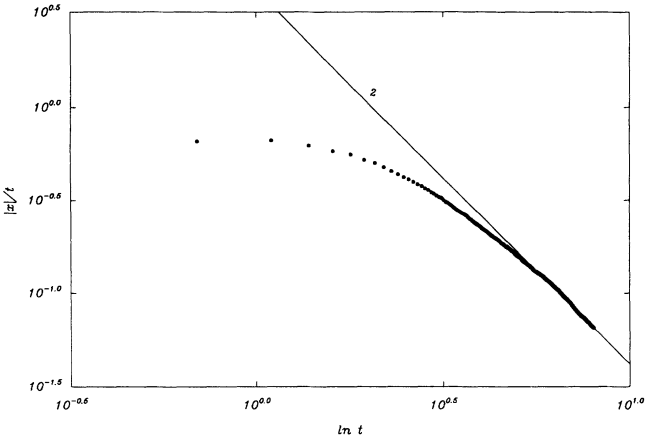

FIG. 2. Scaling plot motivated by the assumption that the disorder-induced wandering of the DPRM at finite, but nonzero p/c is sub-ballistic and controlled by a new fixed point. A least squares fit to the data yields $\psi = 2.02$, in agreement with the Flory argument discussed in the text. This result is similar to but nevertheless distinct from that found for the purely columnar problem at finite temperature, where $\psi = 3$ [13].

which is nicely confirmed by the data in Fig. 2, establishing our belief that wandering of the p/c DPRM is truly *sub-ballistic* in nature and of a different universality class than purely point or columnar problems. For the 2+1 p/c DPRM, we have $\tau \approx 1.20$, so that $\psi \approx 1.60$. Note the result $\psi = 1 + 2/d$ for the finite-temperature, purely columnar DPRM [13] is recovered by setting τ to its thermal value $\tau = 2$ [17].

Lastly, in Fig. 3, we show globally optimal paths through the random energy landscape for various values of p/c, which are indicative of the configurations of the directed polymer that result from the competing effects of point and columnar pins. For $p/c = 0$ the trajectory is absolutely straight as the polymer becomes strictly localized on the nearest favorable columnar defect, while for small, though nonzero $p/c = 0.1$, 0.2, 0.5, the directed polymer exhibits small amplitude "zero-point motion" about the minimal energy pin. This effect is exacerbated

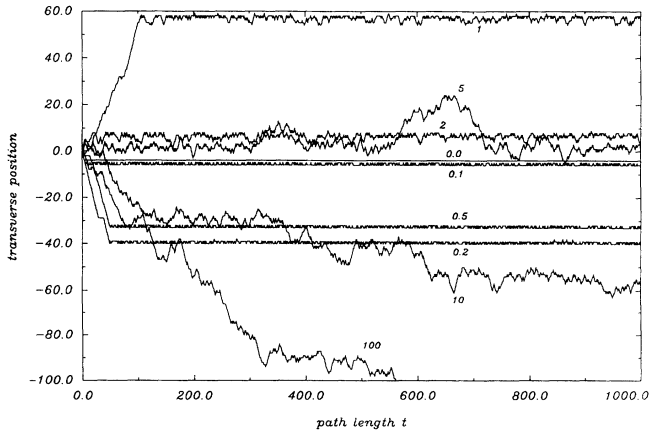

FIG. 3. Globally optimal trajectories of directed polymers in random media subject to point versus columnar defects for various values of p/c.

as the relative strength of point versus columnar defects is augmented to small integer values (note particularly, $p/c = 1$ and 2), the localizing effect of the columnar pin becoming essentially absent when $p/c > 5$. Indeed, there is little qualitative difference between the trajectories for $p/c = 10$ and 100, both being highly reminiscent of globally optimal paths in the standard, purely point disorder DPRM, though we expect, of course, crossover to sub-ballistic wandering at sufficiently large scales.

In conclusion, we have introduced the p/c DPRM, relevant to the disorder-induced roughening of *isolated* vortex lines in ceramic superconducting materials populated by a high density of point and columnar pins. The essential features of our transfer matrix analysis in 1+1 dimensions indicate that DPRM wandering in a mixed random medium, consisting of both point and columnar defects, is truly *sub-ballistic* in nature, sharing many features with the purely columnar DPRM at finite temperature. Nevertheless, forcing the data at finite p/c into a power-law fit leads to an effective wandering exponent, ≈ 0.74, similar to, but exceeding that at the purely point DPRM, where it is $\frac{2}{3}$, and considerably larger than that of the strictly columnar fixed point, characterized by $\zeta = \frac{1}{2}$. Despite the instability of the latter fixed point to the presence of even a small amount of point disorder, a vestigial link to the purely columnar problem is apparent in the near equality of the effective positional and energy fluctuation exponents that we observe for the 1+1 p/c DPRM. Finally, we mention our preliminary findings for the 2+1 p/c DPRM, which suggest the persistence of all these effects in higher dimensions. Using somewhat smaller system sizes and shorter polymers (500 steps), we find $\theta_{2+1} \approx 0.33, 0.68, 0.23$ and $\zeta_{2+1} \approx 0.33, 0.61, 0.62$ for $p/c = 0, 1, 100$, respectively. Again, as advertised, the purely columnar problem has $\theta = \zeta$; in fact, in d dimensions, extremal statistics dictates that the common value is $1/d$. Nevertheless, it must be stressed that the estimates in the mixed case, $p/c = 1$, are truly *effective* exponents. Going to longer polymers will lead to larger values. Indeed, for polymers of length ≈ 2000 steps, we anticipate an increase of roughly 10% in the scaling indices. (A detailed reading of the Civale *et al.* paper [9] reveals an aspect ratio of 2000:1, the mean separation between columnar defects being about 100 Å in their 20-μm-thick sample.) Thicker samples would engender even greater values of ζ_{eff}, pushing the exponent upward toward unity. We note that a large effective wandering exponent in the presence of both point and columnar defects may incur an anomalous signature in the vortex line density as the Meissner phase is approached, by virtue of a simple Flory argument [2] which predicts this quantity to vanish as $(H - H_{c1})^{\zeta/(1-\zeta)}$ in this dimensionality. Indeed, extending the argument to sub-ballistic wandering yields an essential singularity in the vortex line density $n \sim \exp[-(H - H_{c1})^{-1/(2\psi)}]$. Surprisingly, work by Nattermann, Feigelman, and Lyuksyutov [18] has recently called this naive Flory argument into question for point defects, suggesting that the vortex line density vanish *linearly* for such disorder-induced fluctuations, the same as for thermal roughening, where $\zeta = \frac{1}{2}$. In any case, it is clear that high-T_c experimentalists would do well to determine the vortex number density as a function of field near H_{c1}, using Bitter-pattern techniques atop pure, point and columnar disordered samples. This simple measurement might go far to resolve a number of thorny theoretical issues.

Note added in proof. Readers should be aware of complementary work [19] regarding DPRM delocalization transitions exhibited in the presence of point disorder and a *single* columnar defect.

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